Measuring Unfair Inequality: Reconciling Equality of Opportunity and Freedom from Poverty

Paul Hufe*, **Ravi Kanbur**† **& Andreas Peichl**
(*University of Munich; †Cornell University)

Cornell Labour Economics Workshop

Ithaca, 2018-11-19

- Let $Y^e = \{y_1^e, y_2^e, ..., y_n^e\}$ be the empirical distribution of income.
- Let the mean of the distribution be μ.
- Consider standard measures of inequality:

$$G = \frac{1}{n}\left(n + 1 - 2\frac{\sum_{i=1}^{n}(n+1-i)y_i^e}{\sum_{i=1}^{n} y_i^e}\right)$$

$$A(\epsilon) = \begin{cases} 1, & \epsilon = 0 \\ 1 - \frac{1}{\mu}\left(\prod_{i=1}^{n} y_i^e\right)^{1/n}, & \epsilon = 1 \\ 1 - \frac{1}{\mu}\left(\frac{1}{n}\sum_{i=1}^{n}(y_i^e)^{1-\epsilon}\right)^{1/(1-\epsilon)}, & \text{otherwise.} \end{cases}$$

$$\text{MLD} = \frac{1}{n}\sum_{i=1}^{n} \ln \frac{\mu}{y_i^e}$$

Each of these measures can in turn be seen as a divergence metric between the vector of observed incomes

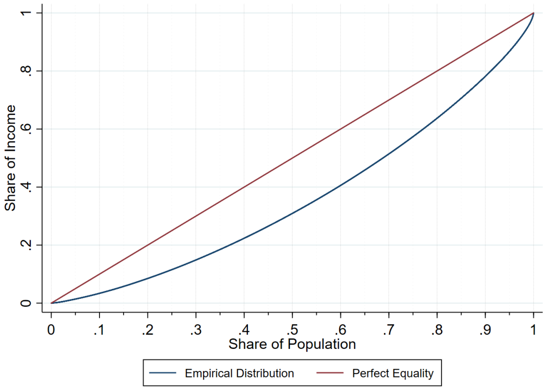
$$Y^e = \{y_1^e, y_2^e, ..., y_n^e\}, \quad (1)$$

and the vector where each element is μ

$$M = \{\mu, \mu, ..., \mu\} \quad (2)$$

In other words, the vector where total income is distributed equally.

Figure: Lorenz-Curve Representation



- In the conventional inequality measurement literature, all the action resides in the properties of this divergence metric.
- Desirable properties for this metric include:
  - Scale Independence
  - Principle of Populations
  - Pigou-Dalton Principle of Transfers

- An additional property often used is sub-group decomposability. This property, with a few other assumptions leads to the Generalized Entropy class of inequality measures:

$$\mathrm{GE}(\alpha) = \begin{cases} \frac{1}{n}\sum_{i=1}^{n} \ln\left(\frac{\mu}{y_i^e}\right), & \alpha = 0 \\ \frac{1}{n}\sum_{i=1}^{n} \left(\frac{y_i^e}{\mu}\right) \ln\left(\frac{y_i^e}{\mu}\right), & \alpha = 1 \\ \frac{1}{n}\frac{1}{\alpha(\alpha-1)}\sum_{i=1}^{n}\left[\left(\frac{y_i^e}{\mu}\right)^{\alpha} - 1\right], & \text{otherwise.} \end{cases} \tag{3}$$

- Note that with $\alpha = 0$ we have the MLD measure.

- Note that this is all to do with measuring inequality. It is a pure distributional question.
- Of course if we move to redistribute then there will be incentive effects and the mean will be affected. This leads to the large literature on optimum taxation, going back to Mirrlees (1971).
- This will NOT be the focus of this talk.

- All of the above is to do with the divergence metric between the observed distribution Y^e and the reference distribution M, the perfect equality distribution.
- But a resurgent, perhaps insurgent, part of the literature argues that what is at issue is not so much the metric of divergence of the actual from the reference vector, but the reference distribution itself.
- Why should we take equality of outcomes as the reference, or the norm, or, in effect, the ideal? Surely the process whereby the outcomes came to be, matters as well?

The general problem is then posed as the divergence between the observed distribution Y^e and a reference or a norm distribution

$$Y^r = \{y_1^r, y_2^r, ..., y_n^r\} \tag{4}$$

Y^r has the same mean as the observed distribution but is not necessarily M, the perfect equality distribution.

Our Contribution

We will focus on two well-established principles of distributive justice (Konow, 2003; Konow and Schwettmann, 2016), namely ...

- Equality of Opportunity (EOp)
- Freedom from Poverty (FfP)

... to derive a new empirical measure for unfair inequalities.

Outline

Outline

Equality of Opportunity

- The primary question concerns the construction of the norm vector.
- This is where the insurgency in the inequality measurement literature has come in recent years.
- The insurgency's premise is that what matters normatively is not equality of outcome, but equality of opportunity.
- This insurgency has deep roots in an older and esteemed philosophical literature.

- Metaphors associated with this view are "leveling the playing field" and "starting gate equality".
- Main philosophical accounts:
  1. Rawls (1971): Fair chance to achieve positions.
  2. Dworkin (1981a,b): Resource egalitarianism.
  3. Arneson (1989): Equal Opportunity for Welfare.
  4. Cohen (1989): Equal Access to Advantage.

In general, Equality of Opportunity pre-supposes that all determinants of individual outcomes are the result of two sets of factors:

1. Circumstances, $C \in \Omega$: Factors beyond individual control.
   $\rightarrow$ Unfair
2. Efforts, $E \in \Theta$: Factors within the control of individuals.
   $\rightarrow$ Fair

Figure: Chetty et al. (2014)

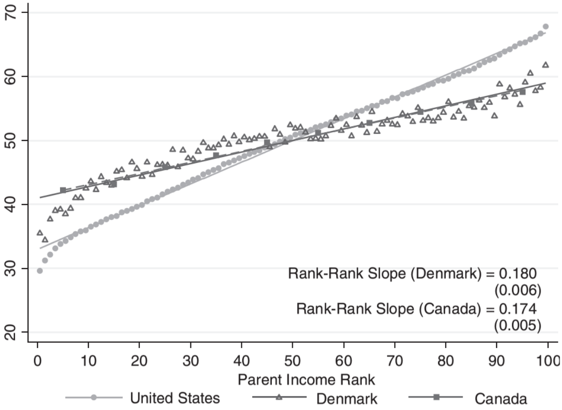
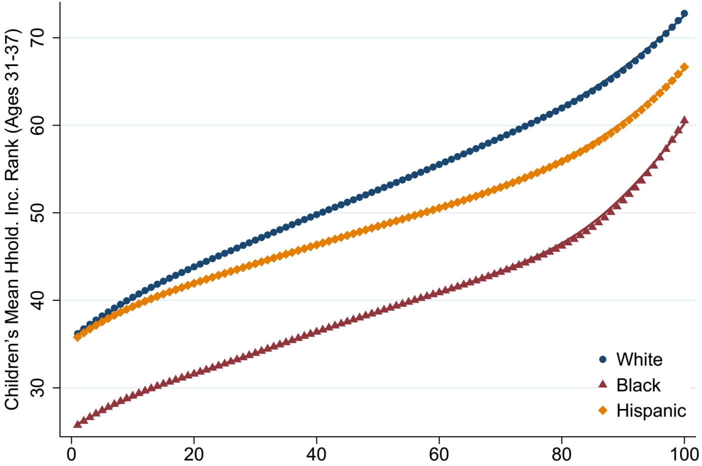


Figure: Chetty et al. (2018)



Based on these circumstances we can partition the population into types:

	High Parental Inc.	Low Parental Inc.
White	Type 1	Type 3
Non-White	Type 2	Type 4

$\rightarrow$ Inequality between circumstance types is morally objectionable.

The equality of opportunity principle is reflected in distributional preferences:

- Vignette studies: Faravelli (2007).
- Survey Experiments: Alesina et al. (2018).
- Lab Experiments: Cappelen et al. (2007); Krawczyk (2010); Mollerstrom et al. (2015).

Freedom from Poverty

- Are ex-post inequalities a matter of indifference for fairness evaluations?
- Some answers:
  - Fleurbaey (1995, 2008) argues for outcome egalitarianism in spheres of social interest -> satisfaction of basic needs.
  - Anderson (1999) argues against pure opportunity egalitarians based on a number of examples -> abandonment of negligent victims.

Based on realized outcomes we can partition the population into groups where $P = \{i : y_i \leq y_{\min}\}$ and $R = \{i : y_i > y_{\min}\}$:

	P	R
L	$y_i \leq \mu_P$	$y_i \leq \mu_R$
H	$y_j > \mu_P$	$y_i > \mu_R$

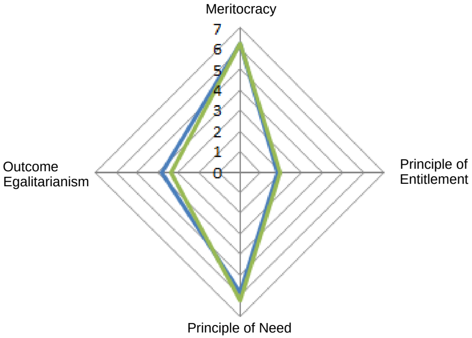
$\rightarrow$ Inequalities are objectionable (i) among individuals in P and (ii) to the extent that $\mu_P < y_{\min}$.

The freedom from poverty principle is reflected in distributional preferences:

- Vignette studies: Gaertner and Schwettmann (2007); Konow (2001).
- Lab Experiments: Cappelen et al. (2013).

Recent evidence from Germany (Sep 2018):

Figure: Eisnecker et al. 2018



Outline

Figure: Norm-Based Inequality Measurement

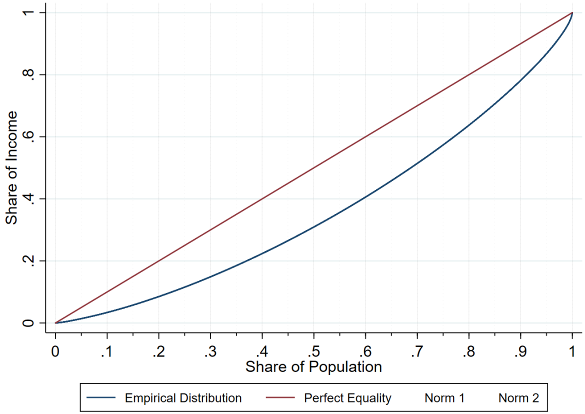
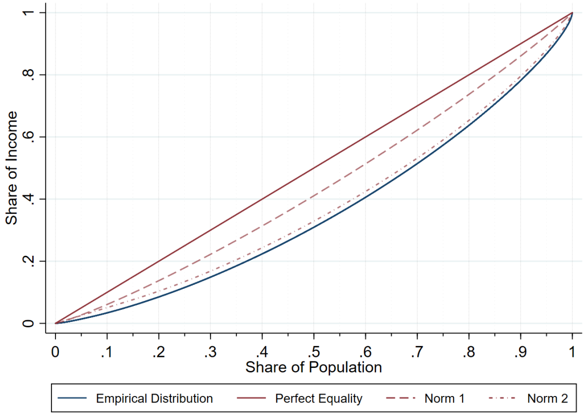


Figure: Norm-Based Inequality Measurement



Norm Vector

Consider the following restrictions on the set of all possible income distributions D:

- Constant Resources:

$$D^1 = \left\{ D : \sum_i y_i^r = \sum_i y_i^e \right\} \tag{5}$$

- Equality of Opportunity:

$$D^2 = \{ D : \mu_t^r = \mu \ \forall \ t \in T \} \tag{6}$$

Consider the following restrictions on the set of all possible income distributions D:

- Constant Resources:

$$D^1 = \left\{ D : \sum_i y_i^r = \sum_i y_i^e \right\} \tag{5}$$

- Equality of Opportunity:

$$D^2 = \{ D : \mu_t^r = \mu \; \forall \; t \in T \} \tag{6}$$

- Freedom from Poverty:

$$D^3 = \{D : y_i^r = y_{\min} \ \forall \ i \in P\} \tag{7}$$

- Financing I:

$$D^4 = \{D : y_i^r \geq y_{\min} \ \forall \ i \in R\} \tag{8}$$

- Financing II:

$$D^5 = \left\{D : \forall t \in T, \ \frac{y_i^r - y_{\min}}{y_j^r - y_{\min}} = \frac{y_i^e - y_{\min}}{y_j^e - y_{\min}} \ \forall \ i, j \in t \cap R\right\} \tag{9}$$

- Freedom from Poverty:

$$D^3 = \{D : y_i^r = y_{\min} \ \forall \ i \in P\} \tag{7}$$

- Financing I:

$$D^4 = \{D : y_i^r \geq y_{\min} \ \forall \ i \in R\} \tag{8}$$

- Financing II:

$$D^5 = \left\{D : \forall t \in T, \ \frac{y_i^r - y_{\min}}{y_j^r - y_{\min}} = \frac{y_i^e - y_{\min}}{y_j^e - y_{\min}} \ \forall \ i, j \in t \cap R\right\} \tag{9}$$

- Freedom from Poverty:

$$D^3 = \{D : y_i^r = y_{\min} \ \forall \ i \in P\} \tag{7}$$

- Financing I:

$$D^4 = \{D : y_i^r \geq y_{\min} \ \forall \ i \in R\} \tag{8}$$

- Financing II:

$$D^5 = \left\{D : \forall t \in T, \ \frac{y_i^r - y_{\min}}{y_j^r - y_{\min}} = \frac{y_i^e - y_{\min}}{y_j^e - y_{\min}} \ \forall \ i, j \in t \cap R\right\} \tag{9}$$

The intersection $\cap_{s=1}^{5} D^s$ yields a singleton:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e < y_{\min} \\ y_i^e \left[1 - \tilde{y}_i \left(\tau^{\text{FfP}} + \tau^{\text{EOp}}(1 - \tau^{\text{FfP}})\right)\right], & \text{otherwise.} \end{cases} \tag{10}$$

where

$$\tilde{y}_i = \left(\frac{y_i^e - y_{\min}}{y_{\min}}\right),$$

$$\tau^{\text{FfP}} = \frac{N_P(y_{\min} - \mu_P^e)}{N_R(\mu_R^e - y_{\min})},$$

$$\tau_t^{\text{EOp}} = \frac{\mu_t^e + \frac{N_{P\cap t}}{N_t}(y_{\min} - \mu_{P\cap t}^e) - \tau^{\text{FfP}}(\frac{N_{R\cap t}}{N_t}(\mu_{R\cap t}^e - y_{\min})) - \mu}{\mu_t^e + \frac{N_{P\cap t}}{N_t}(y_{\min} - \mu_{P\cap t}^e) - \tau^{\text{FfP}}(\frac{N_{R\cap t}}{N_t}(\mu_{R\cap t}^e - y_{\min})) - y_{\min}}$$

Divergence Measure

- Unfair inequality is then measured as the divergence $D(Y^e||Y^r)$ between the observed and the norm income distribution.
- Various divergence measures have been proposed in the literature: Almås et al. (2011); Cowell (1985); Magdalou and Nock (2011).
- We rely on a generalization of the generalized entropy class proposed by Magdalou and Nock (2011) with $\alpha = 0$:

$$D(Y^e||Y^r) = \frac{1}{N}\sum_i \left[\ln \frac{y_i^r}{y_i^e} + \frac{y_i^e}{y_i^r} - 1\right]. \qquad (11)$$

Properties

Imagine we are indifferent to FfP. Then, the norm vector simplifies to:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e < y_{\min} \\ y_i^e \left[1 - \tilde{y}_i \left(\tau^{\text{FfP}} + \tau^{\text{EOp}}(1 - \tau^{\text{FfP}})\right)\right], & \text{otherwise.} \end{cases}$$

$$= y_i^e \left[1 - \left(\frac{\mu_t^e - \mu}{\mu_t^e}\right)\right] = y_i^e \left[\frac{\mu}{\mu_t^e}\right]$$

Using $y_i^e \left[\frac{\mu}{\mu_t^e}\right]$ in the measure of distributional change gives:

$$D(Y^e || Y^r_{\text{EOp}}) = \frac{1}{N} \sum_i \left[\ln \frac{y_i^r}{y_i^e} + \frac{y_i^e}{y_i^r} - 1\right]$$
$$= \frac{1}{N} \sum_i \ln \frac{\mu}{\mu_t^e}.$$

This is a summary statistic of the distribution of type income means: the mean log deviation.

Imagine we are indifferent to EOp. Then, the norm vector simplifies to:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e < y_{\min} \\ y_i^e \left[1 - \tilde{y}_i \left(\tau^{\text{FfP}} + \tau^{\text{EOp}}(1 - \tau^{\text{FfP}})\right)\right], & \text{otherwise.} \end{cases}$$

$$= \begin{cases} y_{\min}, & \text{if } y_i^e < y_{\min} \\ y_i^e \left[1 - \tilde{y}_i \tau^{\text{FfP}}\right], & \text{otherwise.} \end{cases}$$

Using this norm vector in the measure of distributional change gives:

$$D(Y^e || Y^r_{\text{FfP}}) = \underbrace{\frac{1}{N}\sum_{i \in P} \ln \frac{y_{\min}}{y_i^e}}_{\text{Watts Index}} - \underbrace{\frac{1}{N}\sum_{i \in P} \left(\frac{y_{\min} - y_i^e}{y_{\min}}\right)}_{\text{Poverty Gap}} + \frac{1}{N}\sum_{i \in R} \ln \frac{y_i^r}{y_i^e} + \left(\frac{y_i^e}{y_i^r} - 1\right).$$

This is incorporates two widely used poverty measures, the Watts Index and the Poverty Gap ratio.

Outline

Methods and Data

Data:

- Cross-sectional: EU-SILC 2011.
- Longitudinal: PSID (1969-2012).

Type Partition:

- Circumstances: Sex, Occupation Parents, Education Parents, Immigration Background (Race) (36 types).

Income Concept:

- Equivalized disposable HH income (OECD equivalence scale).

Poverty Measure:

- At-Risk-Of-Poverty-Rate (60% of median income).

Data:

- Cross-sectional: EU-SILC 2011.
- Longitudinal: PSID (1969-2012).

Type Partition:

- Circumstances: Sex, Occupation Parents, Education Parents, Immigration Background (Race) (36 types).

Income Concept:

- Equivalized disposable HH income (OECD equivalence scale).

Poverty Measure:

- At-Risk-Of-Poverty-Rate (60% of median income).

Data:

- Cross-sectional: EU-SILC 2011.
- Longitudinal: PSID (1969-2012).

Type Partition:

- Circumstances: Sex, Occupation Parents, Education Parents, Immigration Background (Race) (36 types).

Income Concept:

- Equivalized disposable HH income (OECD equivalence scale).

Poverty Measure:

- At-Risk-Of-Poverty-Rate (60% of median income).

Data:

- Cross-sectional: EU-SILC 2011.
- Longitudinal: PSID (1969-2012).

Type Partition:

- Circumstances: Sex, Occupation Parents, Education Parents, Immigration Background (Race) (36 types).

Income Concept:

- Equivalized disposable HH income (OECD equivalence scale).

Poverty Measure:

- At-Risk-Of-Poverty-Rate (60% of median income).

Results

Figure: Unfair Inequality by Country (Europe)

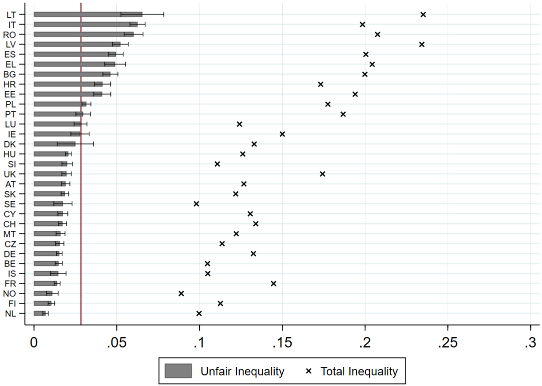


Figure: Unfair Inequality over Time (USA)

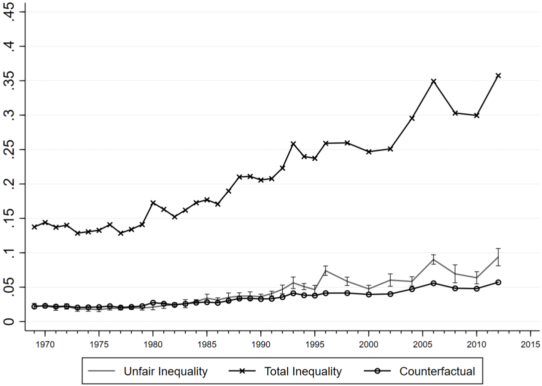


Figure: Decomposition by Country (Europe)

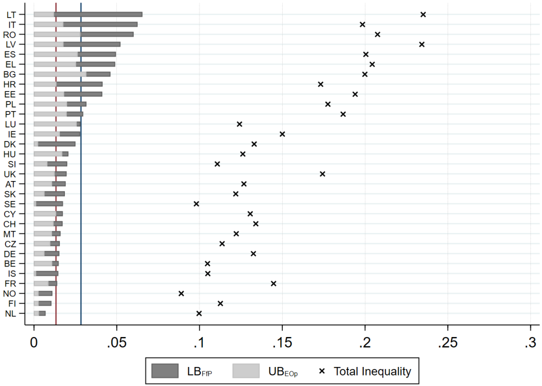
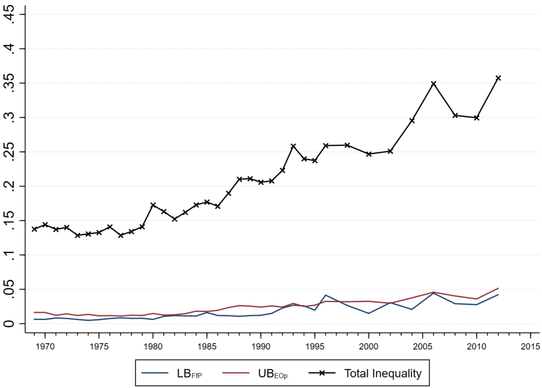


Figure: Decomposition over Time (USA)



Sensitivity Checks

1. Varying poverty thresholds. Graph
2. Alternations in normative assumptions. Graph
3. Alternative divergence measures: Almås et al. (2011); Cowell (1985); Magdalou and Nock (2011). Table

Outline

1. The extent of unfairness/inequity in observed inequality is either overstated (standard inequality measures) or understated (EOp measures).
2. We recognize the multiplicity of fairness ideals by drawing onto the principles of EOp and FfP.
3. Combining different normative principles, i.e. EOp and FfP, yields strong upwards corrections of the unfair share of inequality.
4. The framework may be fruitfully complemented by further ideals of fairness.

Q&A

Thank you!

sk145@cornell.edu
hufe@ifo.de

References I

Alesina, A., Stantcheva, S., and Teso, E. (2018). Intergenerational mobility and preferences for redistribution. *American Economic Review*, 108(2):521–554.

Almås, I., Cappelen, A. W., Lind, J. T., Sørensen, E. Ø., and Tungodden, B. (2011). Measuring unfair (in)equality. *Journal of Public Economics*, 95:488–499.

Anderson, E. S. (1999). What is the point of equality? *Ethics*, 109(2):287–337.

Arneson, R. J. (1989). Equality and equal opportunity for welfare. *Philosophical Studies*, 56(1):77–93.

Cappelen, A. W., Hole, A. D., Sørensen, E. Ø., and Tungodden, B. (2007). The Pluralism of Fairness Ideals: An Experimental Approach. *American Economic Review*, 97(3):818–827.

Cappelen, A. W., Konow, J., , Sørensen, E. Ø., and Tungodden, B. (2013). Just Luck: An Experimental Study of Risk-Taking and Fairness. *American Economic Review*, 103:1398–1413.

Chetty, R., Friedman, J. N., Hendren, N., Jones, M. R., and Porter, S. R. (2018). The opportunity atlas: Mapping the childhood roots of social mobility. *National Bureau of Economic Research Working Paper Series*, 25147.

Chetty, R., Friedman, J. N., Leth-Petersen, S., Nielsen, T. H., and Olsen, T. (2014). Active vs. Passive Decisions and Crowd-Out in Retirement Savings Accounts: Evidence from Denmark. *Quarterly Journal of Economics*, 129(3):1141–1219.

Cohen, G. A. (1989). On the Currency of Egalitarian Justice. *Ethics*, 99(4):906–944.

Cowell, F. A. (1985). Measures of distributional change: An axiomatic approach. *Review of Economic Studies*, 52(1):135–151.

Dworkin, R. (1981a). What is Equality? Part 1: Equality of Welfare. *Philosophy & Public Affairs*, 10(3):185–246.

Dworkin, R. (1981b). What is Equality? Part 2: Equality of Resources. *Philosophy & Public Affairs*, 10(4):283–345.

Faravelli, M. (2007). How context matters: A survey based experiment on distributive justice. *Journal of Public Economics*, 91(7–8):1399–1422.

Fleurbaey, M. (1995). Equal opportunity or equal social outcome? *Economics and Philosophy*, 11(1):25–55.

Fleurbaey, M. (2008). *Fairness, Responsibility, and Welfare*. Oxford University Press, Oxford.

References II

Gaertner, W. and Schwettmann, L. (2007). Equity, responsibility and the cultural dimension. *Economica*, 74(296):627–649.

Konow, J. (2001). Fair and square: The four sides of distributive justice. *Journal of Economic Behavior & Organization*, 46:137–164.

Konow, J. (2003). Which Is the Fairest One of All? A Positive Analysis of Justice Theories. *Journal of Economic Literature*, 41(4):1188–1239.

Konow, J. and Schwettmann, L. (2016). The economics of justice. In Sabbagh, C. and Schmitt, M., editors, *Handbook of Social Justice Theory and Research*, chapter 5, pages 83–106. Springer, New York.

Krawczyk, M. (2010). A glimpse through the veil of ignorance: Equality of opportunity and support for redistribution. *Journal of Public Economics*, 94:131–141.

Magdalou, B. and Nock, R. (2011). Income distributions and decomposable divergence measures. *Journal of Economic Theory*, 146(6):2440–2454.

Mirrlees, J. A. (1971). An Exploration in the Theory of Optimum Income Taxation. *Review of Economic Studies*, 38(2):175–208.

Mollerstrom, J., Reme, B.-A., and Sørensen, E. Ø.. (2015). Luck, choice and responsibility: An experimental study of fairness views. *Journal of Public Economics*, 131:33–40.

Rawls, J. (1971). *A Theory of Justice*. The Belknap Press of Harvard University Press, Cambridge.

Roemer, J. E. and Trannoy, A. (2015). Equality of Opportunity. In Atkinson, A. B. and Bourguignon, F., editors, *Handbook of Income Distribution*, volume 2A, chapter 4, pages 217–300. Elsevier, Amsterdam.

Figure: Alternative Poverty Thresholds by Country (Europe)

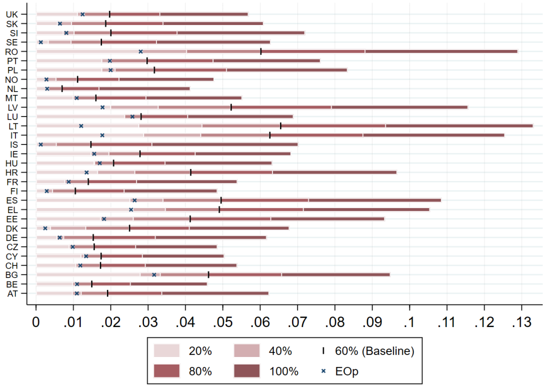
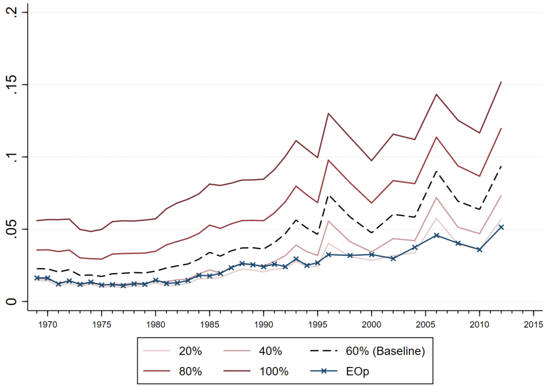


Figure: Alternative Poverty Thresholds over Time (USA)



▸ Back

Figure: Alternative Norm Distributions by Country (Europe)

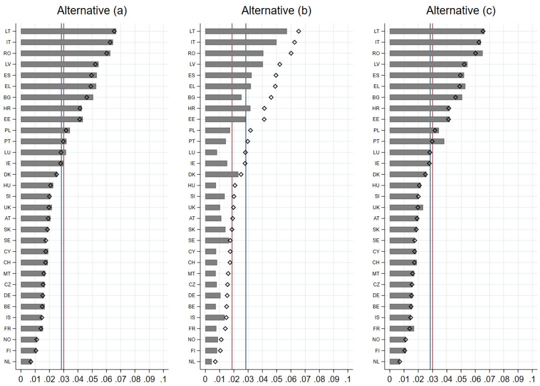
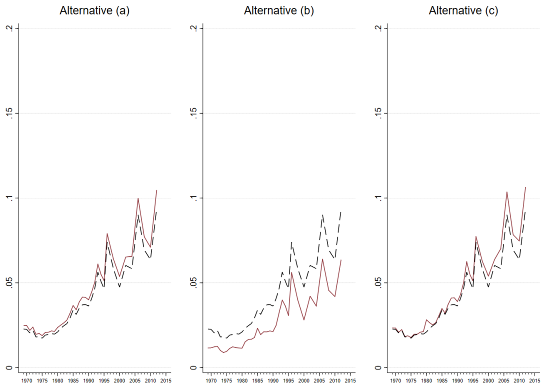


Figure: Alternative Norm Distributions over Time (USA)



Back

Table: Rank Correlation of Measures by Country (Europe)

	Magdalou and Nock			Cowell			Almås et al.
	$\alpha = 0$ (Baseline)	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	
Magdalou and Nock							
$\alpha = 0$	1.000						
$\alpha = 1$	0.953	1.000					
$\alpha = 2$	0.911	0.982	1.000				
Cowell							
$\alpha = 0$	0.975	0.988	0.963	1.000			
$\alpha = 1$	0.953	1.000	0.982	0.988	1.000		
$\alpha = 2$	0.939	0.994	0.986	0.976	0.994	1.000	
Almås et al.							
	0.912	0.970	0.971	0.955	0.970	0.970	1.000

Table: Rank Correlation of Measures over Time (USA)

	Magdalou and Nock			Cowell			Almås et al.
	$\alpha = 0$ (Baseline)	$\alpha = 1$	$\alpha = 2$	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$	
Magdalou and Nock							
$\alpha = 0$	1.000						
$\alpha = 1$	0.991	1.000					
$\alpha = 2$	0.961	0.975	1.000				
Cowell							
$\alpha = 0$	0.994	0.998	0.971	1.000			
$\alpha = 1$	0.991	1.000	0.975	0.998	1.000		
$\alpha = 2$	0.986	0.998	0.979	0.994	0.998	1.000	
Almås et al.							
	0.972	0.984	0.966	0.978	0.984	0.985	1.000

Back